

**Exercise 22**

Evaluate the integral.

$$\int_0^1 (1 - 8v^3 + 16v^7) dv$$

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**Solution**

According to part 2 of the fundamental theorem of calculus,

$$\int_a^b f(x) dx = F(b) - F(a),$$

where  $F$  is an antiderivative of  $f$ . Use the properties of integrals given at the bottom of page 385 to simplify the integral before using this theorem.

$$\begin{aligned} \int_0^1 (1 - 8v^3 + 16v^7) dv &= \int_0^1 1 dv - \int_0^1 8v^3 dv + \int_0^1 16v^7 dv \\ &= \int_0^1 v^0 dv - 8 \int_0^1 v^3 dv + 16 \int_0^1 v^7 dv \\ &= \left( \frac{v^1}{1} \right) \Big|_0^1 - 8 \left( \frac{v^4}{4} \right) \Big|_0^1 + 16 \left( \frac{v^8}{8} \right) \Big|_0^1 \\ &= \left( \frac{1^1}{1} - \frac{0^1}{1} \right) - 8 \left( \frac{1^4}{4} - \frac{0^4}{4} \right) + 16 \left( \frac{1^8}{8} - \frac{0^8}{8} \right) \\ &= 1 - 8 \left( \frac{1}{4} \right) + 16 \left( \frac{1}{8} \right) \\ &= 1 \end{aligned}$$